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Erratum

Erratum to "Global heat kernel bounds via desingularizing weights"☆ [J. Funct. Anal. 212 (2004) 373–398]

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The publisher regrets that the article is missing approximately one page of the original manuscript (on page 395 between lines 10 and 9, counting from the bottom of the page). Also, the statement of Corollary 4 of the article contains a line (on page 378 line 13 counting from the bottom of the page) which was not in the original manuscript and is completely irrelevant. The missing text is given here.

Let us introduce Nash's G-function by

$$G(t) \equiv G(t,z) := \langle e_0(\cdot) \log[\varepsilon + k(t,z,\cdot)] \rangle, \quad 0 < t \leqslant s/\delta, \quad \varepsilon > 0$$

and suppose that the following estimate

$$G(t,z) \geqslant -\tilde{Q}(t) + \log \varphi_s(z) - \tilde{C} \qquad (G\text{-bound})$$

holds for all $t \in [s/2\delta, s/\delta]$ and $z \in B_{\sqrt{t}}(0)$ with constants $\tilde{C}$ and $\delta \geqslant 1$ depending only on d, β. By the G-bound, $\langle e_0(\cdot) \log k(t,z,\cdot) \rangle \geqslant -\tilde{Q}(t) + \log \varphi_s(z) - \tilde{C}$. Thus,

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taking into account that $\log\langle e\rangle = (d/2)\log 4\pi + \tilde{\mathcal{Q}}(s) \geqslant (d/2)\log 4\pi\delta + \tilde{\mathcal{Q}}(t)$, we have

$$\log k(2t, x, y) \geqslant -\tilde{\mathcal{Q}}(t) + \log \varphi_s(x) + \log \varphi_s(y) - 2\tilde{C} + (d/2)\log 4\pi\delta$$

or

$$k(2t, x, y) \geqslant ct^{-d/2}\varphi_s(x)\varphi_s(y), \quad c = c(d, \beta) > 0, \quad x, y \in B_{\sqrt{t}}(0).$$

The latter is equivalent to (24).

Derivation of the G-bound: First we note that

$$G' \equiv \frac{d}{dt}G(t, z) = \left\langle e_0(\cdot)\,\frac{k'(t, z, \cdot)}{\varepsilon + k(t, z, \cdot)}\right\rangle = -\left\langle \frac{e_0}{\varepsilon + k}\,(-\Delta + (\Delta\varphi/\varphi))k\right\rangle$$
$$= -\left\langle e_0(\Delta\varphi/\varphi)\,\frac{k}{\varepsilon + k}\right\rangle - \left\langle \nabla\,\frac{e_0}{\varepsilon + k}, \nabla k\right\rangle \equiv I_1 + I_2.$$

Let us estimate I_1, I_2 from below as follows.

$I_1 \geqslant \langle e_0 V\,\frac{k}{\varepsilon+k}\rangle - \frac{C}{s}\langle e_0\,\frac{k}{\varepsilon+k}\rangle \geqslant \langle e_0 V\rangle - C/s$, where the constant C stems from the definition of φ. The term $\langle e_0 V\rangle$ plays no role even though it is $\geqslant 0$ because, due to the Hardy inequality, $\langle e_0 V\rangle = \beta\langle\sqrt{e_0}V_0\sqrt{e_0}\rangle \leqslant \beta\langle(\nabla\sqrt{e_0})^2\rangle \leqslant C/s$, $C = C(d)$.

$I_2 = -\langle e_0\nabla\log e_0, \nabla\log(\varepsilon + k)\rangle + \langle e_0|\nabla\log(\varepsilon + k)|^2\rangle$. By quadratic estimates,

$$2I_2 \geqslant -\langle(\nabla e_0)^2/e_0\rangle + \langle e_0|\nabla\log(\varepsilon + k)|^2\rangle \geqslant -C/s + \langle e_0|\nabla\log(\varepsilon + k)|^2\rangle, \quad C = C(d).$$

Thus $G' \geqslant -\frac{C_0}{s} + \frac{1}{2}\langle e_0|\nabla\log(\varepsilon + k)|^2\rangle$, $C_0 = C_0(d)$. Since $C_0/s \leqslant C_0/\delta t$, we make the first choice of $\delta : \delta \geqslant 2C_0/d$, obtaining

$$(G + \tilde{\mathcal{Q}})'(t) \geqslant \tfrac{1}{2}\langle e_0|\nabla\log(\varepsilon + k)|^2\rangle.$$

At this point we employ the spectral gap inequality

$$\langle e_0|\nabla f|^2\rangle \geqslant \frac{1}{2s}\,\langle e_0|f - \langle e_0 f\rangle|^2\rangle, \quad f \in L^2(R^d, e_0(y)dy),$$

obtaining

$$(G + \tilde{\mathcal{Q}})' \geqslant \frac{1}{4s}\,\langle e_0|\log(\varepsilon + k) - G|^2\rangle.$$